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Relationship Between the Optical Model and the Resonance Theory*

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The relationship between cross sections and the strength function is discussed. The usually used simple linear relation is shown to be an approximation to the more general theory of Wigner and Eisenbud. With this theory, a consistent and simple behavior of the strength function with respect to the energy and the atomic weight can be obtained. The problems of the proper choice of the boundary condition and the matching radius for the diffuse well are discussed. A quantitative estimate of the effect of the nonlinear terms in a typical nucleus Nb^{93} is presented.

I. INTRODUCTION

THE optical model has been very successful in explaining the behavior of low-energy nucleons scattered by complex nuclei (e.g., the average resonance cross sections of neutrons). Many theories[1–5] have been formulated in the past to express the average cross sections in terms of some energy-independent (over a limited energy range) parameters like the strength function. These theories are called resonance theories. There also have been attempts to indicate the connection between the two, i.e., between the optical model and the resonance theories[3]; however, an exact correspondence between the two has not been checked. In addition, because of the many resonance theories and their modified versions available today, it is not apparently clear which one of them to choose. The most recent of these theories are by Moldauer[4] and Feshbach.[5] So far, their widest use has been in interpreting the low-energy s-wave neutron interactions where all these theories become identical. However, with the improvement in experimental techniques, p-wave neutron-interaction information is becoming available.[6–9] Since this requires reasonably high energies, $\sim$1 to 500 keV, the detailed form of the cross section versus strength-function relation is needed in order to evaluate quantities like p-wave strength functions from the experimentally observed total and partial cross sections.

In Refs. 6–9, a conventional linear relation is assumed between the average compound nucleus cross section and strength function, even though measurements are carried out in the hundreds of kilovolt range. These strength functions are then compared with those obtained from the low-energy ($<$10 keV) optical-model calculations.[10] Therefore, the parameterizations of cross section by the energy-independent quantities (over a limited range), so-called strength functions, should be such so as to be consistent with the optical model.

In the present paper it is shown that the conventional simple linear relationship does not hold at high energies ($\gtrsim$20 keV). Therefore, the author feels that theoretical and experimental quantities are not alike and may lead to wrong conclusions (like the strength of spin-orbit coupling). Besides, one measures the sum contributions of s and p wave in the measurements when the p-wave

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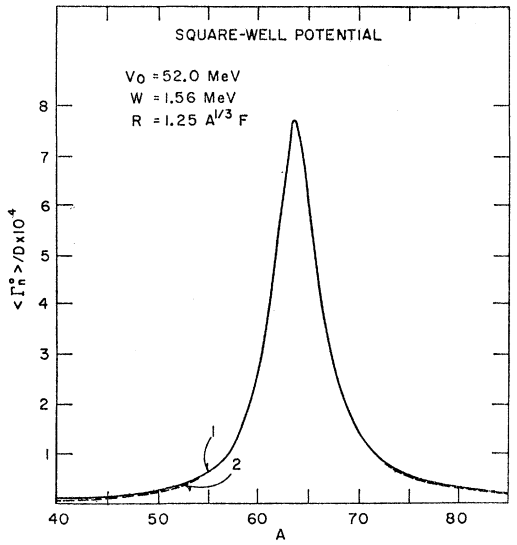

FIG. 1. The S-wave neutron strength function $\langle\Gamma_n^0\rangle/D$ versus atomic weight. Curve 1 is calculated for a square-well optical-model potential with parameters: $V_0=52$ MeV, $W=1.56$ MeV, $R=1.25A^{1/3}$. Curve 2 is the $\langle\Gamma_n^0\rangle/D$ calculated using Eq. (4) and the relation $\langle\Gamma_n^0\rangle/D=RS_0\times10^{-4}/0.2276$.

contribution is appreciable. Because of a different energy dependence of nonlinear terms, it is difficult to find the errors involved, due to the use of a simple linear relationship. There are also disagreements between the various theoretical results.[1–3,5,11] In addition, these deductions are strictly valid for a square-well potential, while the measured strength functions are presently compared with the diffuse surface potential.

Lane and Lynn[12] and Seth[13] have tried to indicate the presence of nonlinear terms in s-wave neutron interactions by a square-well complex potential. However, it is not clear from their work why the conventional linear relation is incorrect. Moreover, the effect for higher partial waves and for the diffuse potential is still an unsolved problem.

Therefore, the present work was carried out to find a relation between the cross section and the strength function such that its energy dependence (for fixed values of strength function) agrees with that obtained from the optical model. Later a discussion of a reasonable choice of boundary condition B_l and the matching radius R for deducing the strength function from the cross section represented by a diffuse potential is given. In the last section the effect of the nonlinear terms on the p-wave strength function from the measured total cross sections in a typical nucleus Nb^{93} is presented.

[11] K. B. Mather and P. Swan, *Nuclear Scattering* (Cambridge University Press, London, 1958), p. 425.

[12] A. M. Lane and J. E. Lynn, Atomic Energy Research Establishment Report No. T/R 2210 (unpublished).

[13] K. K. Seth, Rev. Mod. Phys. **30**, 442 (1958); see also K. K. Seth, Can. J. Phys. **37**, 1199 (1959).

II. THE SQUARE-WELL POTENTIAL

It is easy to see[1,14] that for a square-well complex potential the strength function S_l and the distant level parameter $R_\infty{}^l$ for any partial-wave l satisfy the relations:

$$\pi S_l=\frac{\gamma_\lambda{}^2W}{(E_\lambda-E)^2+W^2}, \tag{1}$$

$$R_\infty{}^l=\frac{\gamma_\lambda{}^2(E_\lambda-E)}{(E_\lambda-E)^2+W^2}, \tag{2}$$

where $\gamma_\lambda{}^2=\hbar^2/mR^2$ is the single-particle width, $W=$ imaginary part of the potential, $E_\lambda=$ energy of the giant resonance, $E=$ energy of the particle, $R=$ nuclear radius, $m=$ mass of the incident particle. Note that the above is true only for an isolated giant resonance λ.

For low-energy neutrons it is easy to see that the compound nucleus cross section for the lth partial wave is given by

$$\sigma_c{}^l\simeq(2l+1)4\pi^2\lambda\!\!\!^{-2}P_lS_l, \tag{3}$$

where P_l is the neutron penetrability. This is in no real disagreement among various formulations.[1–5] The different formulations, however, disagree in their higher order terms which become important only at high energies and near the peaks of the giant resonances. It is easy to verify that the above formula (3) is quite accurate for neutrons below a keV for the imaginary part of the optical-model potential of about 3 MeV or more.

We first verify that (1) satisfies the optical model. For this purpose we use the important property of the optical model that for a fixed W the logarithmic derivative at the surface depends only on the product KR where K is the wave number of the neutron inside the nucleus and R the radius of the target nucleus.[3] This implies that (1) and (2) which exhibit giant-resonance character with respect to energy for a fixed radius can be converted into the ones which show resonance behavior with respect to R for a fixed energy, by the substitution $K_\lambda R=KR_\lambda$. R_λ is the radius at resonance for the case when resonance is obtained by varying R (by changing atomic weight) keeping energy fixed. [This argument holds, strictly speaking, for a square well with a real potential. For the diffuse wells treated here, we expect that Eq. (4) below is a good approximation so long as $V_0\gg W$.] Therefore, for low-energy ($E\ll V_0$), (1) can be written as

$$\pi S_\lambda=\frac{\gamma_\lambda{}^2W}{V_0{}^2[1-R_\lambda{}^2/R^2]^2+W^2}, \tag{4}$$

where V_0 and W are the real and imagniary parts of the potential. This expression allows us to calculate the strength-function giant resonance as a function of the atomic weight or radius for a fixed energy of the incident

[14] A. P. Jain, Ph.D. thesis, Cornell University, 1962 (unpublished).

TABLE I. The calculated values of R_∞^0 and γS_0 for three values of atomic weights: $A=60$ below the resonance peak, $A=64$ at the peak of the resonance, and $A=68$ after the peak of the resonance.

	$A=60$			$A=64$ (res. peak)			$A=68$		
	Correct		Linear approx.	Correct		Linear approx.	Correct		Linear approx.
E (keV)	R_∞^0	πS_0	πS_0	R_∞^0	πS_0	πS_0	R_∞^0	πS_0	πS_0
1	0.535	0.389	0.379	−0.106	1.067	0.995	−0.467	0.337	0.329
501	0.565	0.528	0.248	−0.377	0.918	0.308	−0.426	0.255	0.164

particle in terms of the position of the peak. We can also calculate $\sigma_c{}^l$ from the optical model for a particular set of parameters for the various values of atomic weights at a neutron energy of, say, 1 keV. S_l can then be determined using (3). Figure 1 (curve 1) shows S_0 (for s wave) as calculated using the ABACUS code[15] for a square-well potential and the expression (3). For the sake of convenience we plot a more conventional strength function $\langle\Gamma_n{}^l\rangle/D$ which is related to S_l by the relation $S_l=(0.2276)R^{-1}(\langle\Gamma_n{}^l\rangle/D)\times 10^4$. $\langle\Gamma_n{}^l\rangle$ is the average reduced neutron width and D, the average level spacing. From (4) we also calculate S_l for the value of A_λ (where $R_\lambda=r_0A_\lambda{}^{1/3}$) as obtained by the ABACUS code and W that used in the ABACUS. This is shown by the dotted line (curve 2) in Fig. 1. A good agreement between the two curves indicates the validity of (1) and (2). Similar agreement is found to hold for p waves. The departures in the wings of the resonance arise because (1) and (2) are single-level formulas and in the wings the contribution from nearby giant resonances becomes important. It must be emphasized that this conclusion is independent of the form of the higher order terms in (3) because at 1 keV, or below, the higher order contribution is small for not too low values of W.

Now we show that the Lane and Thomas formulation[1] of the resonance theory with properly chosen boundary condition is the most correct expression of cross section as a function of strength function.

The cross sections can be written as

$$\sigma_c{}^l=\pi\lambda\!\!\!^{-2}(2l+1)T_l=\pi\lambda\!\!\!^{-2}(2l+1)(1-|\eta_l|^2), \quad (5)$$

$$\sigma_{se}{}^l=\pi\lambda\!\!\!^{-2}(2l+1)|1-\eta_l|^2. \quad (6)$$

The average collision function η_l is given by[1]

$$\eta_l=e^{-2i\varphi_l}\begin{bmatrix}\dfrac{1}{R_l}-\hat{L}_l{}^*\\ \dfrac{1}{R_l}-\hat{L}_l\end{bmatrix}, \quad (7)$$

where

$$R_l=R_\infty{}^l+i\pi S_l, \quad (8)$$

$$\hat{L}_l=\hat{\Delta}_l+iP_l, \quad (9)$$

$$\hat{\Delta}_l=\Delta_l-B_l. \quad (10)$$

[15] E. Auerbach (private communication), ABACUS 2, a general purpose optical-model code.

φ_l= the hard-sphere phase shift, $R_\infty{}^l$= the distant level parameter, S_l= the strength function, Δ_l= the shift factor, B_l= the boundary condition parameter, P_l= the penetration factor.

For the time being let us assume that $B_l=-l$. It will be shown later on that this is the only reasonable choice for the boundary condition.

From (7) the compound nucleus cross section for the lth partial wave can be found to be

$$\sigma_c{}^l=\frac{4\pi^2\lambda\!\!\!^{-2}P_lS_l(2l+1)}{(1-\hat{\Delta}_lR_\infty{}^l+\pi P_lS_l)^2+(\hat{\Delta}_l\pi S_l+P_lR_\infty{}^l)^2}. \quad (11)$$

It is easy to verify that for the boundary condition chosen, (11) reduces to (3) in the low-energy limit.

η_l can also be calculated using some optical-model parameters like V_0, W, R, and a, at various energies for a particular radius. Using (7), one can then find R_l (i.e., its real and imaginary parts, $R_\infty{}^l$ and πS_l) at any energy. We now show that these quantities show a variation as expected from (1) and (2). Table I shows $R_\infty{}^l$ and πS^l for a square-well optical-model potential in the range of neutron energy from 1 to 501 keV near the s-wave maximum. The parameters of the potential well are $V_0=52$ MeV, $W=1.56$ MeV, and $R=1.25\ A^{1/3}$F. The calculations are performed for three values of A, one on the resonance and the other two at about half-max, one below and the other above the resonance for the s-wave neutrons. It should be noted that the sign and magnitude of $R_\infty{}^0$ is consistent with (1) and (2), its magnitude being positive below the resonance and negative above the resonance. Moreover, the relative variation in these quantities as a function of energy is also of the right order of mangitude as expected from (1) and (2) and so also is the sign of the variation. For s-wave neutrons $\hat{\Delta}_0=0$, therefore, essentially we test the contributions of the higher order terms in S_l in (11). Also shown are the strength functions assuming (3), called the linear approximation.

Table II shows the calculation for the same set of parameters, but near the p-wave maximum. It is easy to verify that the essential contribution of the next higher order term here is due to $R_\infty{}^l\hat{\Delta}_l$, the effect of πP_lS_l being small for energies below $\frac{1}{2}$ MeV. Again, we see that $R_\infty{}^1$ and πS_1 behave as expected from (1) and (2). On the other hand the strength function obtained, assuming (3) alone, changes rapidly, depending upon

TABLE II. The calculated values of $R_\infty{}^1$ and πS_1 for three values of atomic weights: $A=104$ below the resonance, $A=110$ at the resonance peak, and $A=114$ after the peak of the resonance.

	$A=104$			$A=110$ (res. peak)			$A=114$		
	Correct		Linear approx.	Correct		Linear approx.	Correct		Linear approx.
E (keV)	$R_\infty{}^0$	πS_1	πS_1	$R_\infty{}^1$	πS_1	πS_1	$R_\infty{}^1$	πS_1	πS_1
1	0.399	0.365	0.365	−0.129	0.720	0.720	−0.343	0.370	0.370
501	0.386	0.497	0.413	−0.287	0.590	0.315	−0.328	0.272	0.168

whether one is below or above the resonance energy.

Figure 2 shows the πS_0 and $R_\infty{}^0$ for $A=60$ for the same parameters as in Fig. 1, as a function of energy from 1 keV to 6 MeV. First the amplitudes η_0 are calculated using the optical-model parameters, and next πS_0 and $R_\infty{}^0$ are evaluated for each energy using (7). The full width at half-maximum of curve 1 is 3.12 MeV, as expected from (1). It is easy to verify that the agreement of curves 1 and 2 with that expected from (1) and (2) is very good even up to a few MeV. Similar results are obtained for a nucleus near the p-wave maximum.

This establishes the contribution of the higher order terms in $\langle\Gamma_n\rangle/D=2\pi P_l S_l$ and also in the distant level parameter $R_\infty{}^1$ for p waves, not included by Feshbach[5] in his latest work and missed in the evaluation of p-wave strength functions from the experimentally measured capture[6,7] and total[8] cross sections in the kilovolt range. The use of (3) rather than (11) in the evaluation of strength functions from the known cross sections in the kilovolt range will provide results which will vary rapidly and erratically with atomic weights and energy, unlike the simple variation given by (1) and (2) which provides a smooth variation in a known manner. It also demonstrates that it is incorrect to write the higher order terms as

$$\sigma_c{}^l=(2l+1)\pi\lambda^2(1-e^{-4\pi P_l S_l})$$
$$=(2l+1)\pi\lambda^2(1-e^{-2\pi\langle\Gamma_n\rangle/D}),$$

where $\langle\Gamma_n\rangle$ is the average neutron width, and D is the average level spacing, as speculated by many people.

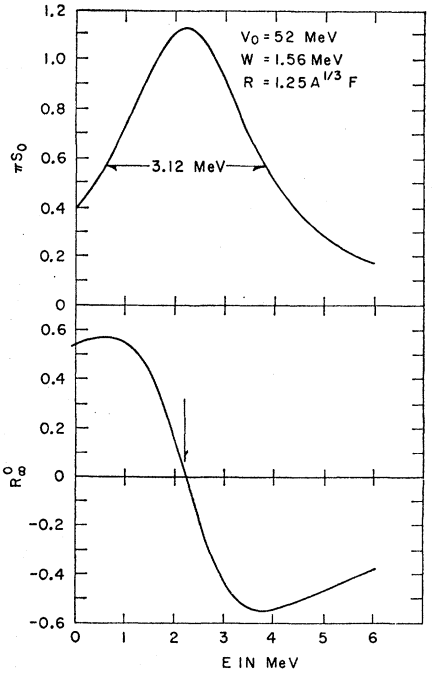

FIG. 2. The strength function πS_0 and the distant level term $R_\infty{}^0$ as a function of energy from 1 keV to 6 MeV for a nucleus with $A=60$. The calculations are for a square-well potential with parameters $V_0=52$ MeV, $W=1.56$ MeV, $R=1.25A^{1/3}$.

Moldauer,[4] in his latest work, does rederive (7) without using the usual channel elimination method, but assumes that $B_l=\Delta_l$ for all partial waves. Such a boundary condition is objectionable because it is energy-dependent. In the energy range of p-wave strength-function measurements (0 to $\sim\frac{1}{2}$ MeV) from the measured average cross sections, the change in B_1, for example, is not negligible. The condition $B_l=\Delta_l$ may be reasonable if one wants to analyze a narrow Breit-Wigner resonance but not when one analyzes average cross sections over a several hundred kilovolt range. We will now show that $B_l=-l$ is the only boundary condition that is a reasonable one. This will be illustrated by an example using p-wave neutrons.

For p-wave neutrons, $\Delta_1=-1/(1+\rho^2)$. With our choice of B_1, $\hat{\Delta}_1=\rho^2/(1+\rho^2)$, where $\rho=kR$, k being the wave number of the neutron outside the nucleus and R the radius. The boundary condition $B_l=-l$ satisfies the criterion that in the low-energy limit ($\rho\to 0$), (5) reduces to (3). However, since B_1 has to be energy-independent, it is easy to verify from (9) that no other choice of B_1 can satisfy this condition. In general, for any partial-wave l, for $B_l=-l$, $\hat{\Delta}_l=\Delta_l-B_l$ is zero in the low-energy limit. Therefore, (3) holds for all partial waves in the low-energy limit. In addition, the simple Eqs. (1) and (2) were derived[14] with the boundary condition $B_l=-l$. For any other condition, the πS_l and $R_\infty{}^l$ are complicated functions of the real part of the optical-model potential, the radius, and the angular momentum l. Moreover, the conventional linear relation between the average compound nucleus cross section and strength function (for all partial waves) is lost even in the low-energy limit, in case any other energy-independent choice for B_l is assumed; i.e., $\hat{\Delta}_l(E=0)\neq 0$

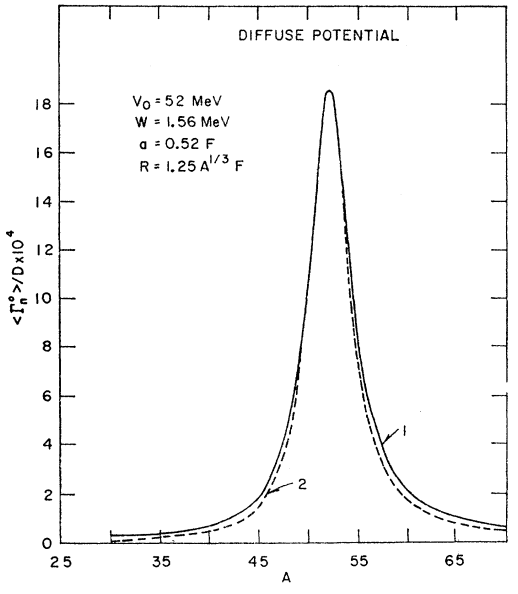

FIG. 3. The S-wave neutron strength function $\langle\Gamma_n^0\rangle/D$ versus atomic weight. Curve 1 is for a diffuse Eckart potential with parameters $V_0=52$ MeV, $W=1.56$ MeV, $a=0.52$ F, $R=1.25$ $A^{1/3}$ F. Curve 2 is the $\langle\Gamma_n^0\rangle/D$ calculated using Eq. (4) and the relation $\langle\Gamma_n^0\rangle/D=RS_0\times10^{-4}/0.2276$. This curve is normalized to the curve 1 at the peak.

means, according to (11), that there will be a factor $[(1-\hat{\Delta}_l R_\infty{}^l)^2+(\hat{\Delta}_l\pi S_l)^2]$ in the denominator. It is for all of these reasons that a boundary condition $B_l=-l$ is preferred over any other.

III. THE DIFFUSE POTENTIAL

We have shown above the connection between the Wigner-Eisenbud resonance theory and the square-well optical-model potential. However, from a study of s-wave strength functions, Hughes *et al.*[16] have demonstrated that the potential is diffuse, like the Eckart type $(V_0+iW)/(1+\exp(r-R)/a)$. Vogt [17,18] has shown that even for the Eckart type of potential, (7) and (11), which were proved to hold for square well, still hold provided $\hat{\Delta}_l$ and P_l are changed to $\hat{\Delta}_l C(a)$ and $P_l C(a)$, respectively, where $C(a)$ is a constant, depending only on the diffuseness a of the potential. From (7) it is easy to see that this is equivalent to changing R_l, or $R_{\infty l}$ and πS_l by $C(a)$ and leaving $\hat{\Delta}_l$ and P_l unchanged. Therefore, if one redefines the $R_\infty{}^l$ and πS_l as $R_\infty{}^l C(a)$ and $\pi S_l C(a)$, then (7) and (11) are still valid with the square-well penetrability P_l and the shift factor $\hat{\Delta}_l$. The corresponding Eqs. (1) and (2) should then also remain unchanged except by a constant factor $C(a)$. We will show that the form of Eqs. (1) and (2) depends considerably on the matching point R for a diffuse potential.

Figure 3 shows the calculations which are the same as those described in Fig. 1, but for a diffuse Eckart potential of diffuseness 0.52 F at a neutron energy of 1 keV. The matching point is taken as R, the half fall-off distance of the potential. The dotted curve which is calculated using (4) is normalized to the solid curve at the peak to remove the effect of the unknown factor $C(a)$ in (1). We notice that the dotted curve has a smaller width at half-max than the solid curve. The same effect is observed for the corresponding p-wave calculations.

Figure 4 shows the calculations of $\langle\Gamma_n^0\rangle/D$ for an atomic weight $A=50$ for the same parameters as above as a function of energy from 1 keV to 6 MeV, for three values of matching radius R, $R+\frac{2}{3}a$, and $R+\frac{4}{3}a$, where R is the half fall-off distance of the potential and a the diffuseness. For all these calculations the amplitude η_0 is evaluated using the ABACUS code and then $\langle\Gamma_n^0\rangle/D$ evaluated using (7) for the above three values of the matching radius. First, all three curves coincide at the lowest energy as they should, because at sufficiently low energy $\sigma_c{}^0\propto\langle\Gamma_n{}^0\rangle/D$ and the R dependence is cancelled out. We notice that for the matching radius or boundary at R (curve 1), the full width at half-max of the curve is about 6.9 MeV. The curve is also asymmetrical about the peak. This is in contrast to what we found for the square-well case; the full width at half-

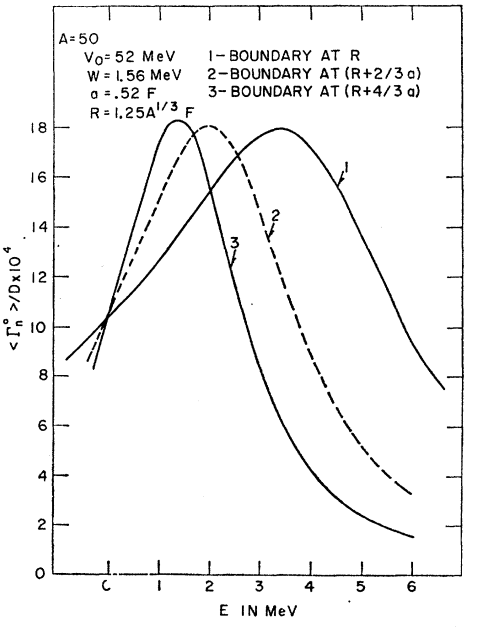

FIG. 4. The S-wave neutron strength functions $\langle\Gamma_n^0\rangle/D$ versus energy from 1 keV to 6 MeV for $A=50$. For all the three curves, the phase shifts are evaluated for an Eckart potential with parameters $V_0=52$ MeV, $W=1.56$ MeV, $a=0.52$ F, and $R=1.25$ $A^{1/3}$ F. $\langle\Gamma_n^0\rangle/D$ in curves 1, 2, and 3 are evaluated for three values of matching radius R, $R+\frac{2}{3}a$ and $R+\frac{4}{3}a$, respectively, in Eq. (8).

[16] D. J. Hughes, R. L. Zimmerman, and R. E. Chrien, Phys. Rev. Letters 1, 461 (1958).
[17] E. Vogt, Phys. Letters 1, 84 (1962).
[18] E. Vogt, Rev. Mod. Phys. 34, 723 (1962).

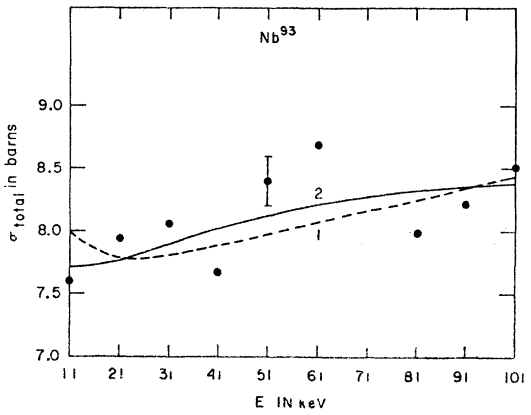

FIG. 5. The dots represent the total cross section of Nb^{93} (Ref. 14) in the range of neutron energies 10 to 100 keV. The solid lines are the Eckart potential calculations with radius $R=1.25\ A^{1/3}$ F and diffuseness $a=0.52$ F. The solid curves 1 and 2 are for the two best-fit parameters with V_0 and W as (50.7, 2.38) and (54.8, 2.10) MeV, respectively.

maximum of curve 1, Fig. 2, is 3.12 MeV. However, this width and asymmetry decrease as the boundary is moved further out. For the boundary $R+\frac{4}{3}a$, the full width is already about 3.1 MeV, very close to the expected value of Eq. (1) of $2W=3.12$ MeV. Similar results are obtained in the case of p-wave strength functions. It is, therefore, concluded that a matching radius of about $R+\frac{4}{3}a$ gives more consistent results than the radius R.

IV. p-WAVE STRENGTH FUNCTION

Recently, considerable interest has arisen in finding p-wave strength functions from the measured capture[6,7] and total cross sections.[8,9,14] Most of the earlier workers[6–8] have used a linear approximation [see Eq. (3)] between the compound nucleus cross section and the strength function for all partial waves. In addition, the shape elastic part of the cross section has been assumed to behave as assumed in Ref. 3, which can be seen from (7) to hold only in the low-energy limit. In this section a quantitative evaluation of the nonlinear terms in a practical case of Nb^{93}, which is near the p-wave giant resonance maximum, is presented.

Figure 5 shows the experimental data[14] for the total cross section of Nb^{93} in the range of 10 to 100 keV. Also shown are the Eckart potential calculations with radius $R=1.25A^{1/3}$F and $a=0.52$F. Curves 1 and 2 are for the two best-fit parameters with V_0 and W as (50.7, 2.38) and (54.8, 2.10) MeV, respectively (the other parameters are kept fixed). These parameters are about the same as used by Campbell *et al.*[19] Both curves are approximately alike and are difficult to distinguish even with very accurate data (~2 to 3%) in the 10- to 100-keV range. Figure 6 (curves 1 and 2) shows the p-wave compound nucleus cross section σ_c^1 calculated, using the same parameters as in curves 1 and 2, respectively, of Fig. 5. The cross section is plotted against $1/(E(eV))^{1/2}\cdot(k^2R^2/k^2R^2+1)$. This is the conventional penetrability for the p-wave neutrons. If the linear approximation were valid, then both the curves 1 and 2 should follow a straight line passing through the origin. The departure from straight lines is a measure of the nonlinear effect. The departure at 100 keV is about 12% for curve 1 and 42% for curve 2. Notice that this departure is in the opposite direction from the straight line in the two cases. This is because of the finite contribution of the distant level term R_∞^1 which assumes equal and opposite value on the two sides of the giant resonance [Eq. (2)]. Similar departures can be seen in the shape elastic cross section which has to be added to the compound nucleus cross section to calculate the total cross section. An even larger effect is observed for the nucleus for which a smaller W is needed to fit the experimental data. In (9) the largest contributing term that depends on R_∞^l is $\hat{\Delta}_l R_\infty^l$ in the denominator. The fact that the nonlinear terms are of opposite sign confirms that $\hat{\Delta}_l=0$ for $l\neq 0$ is not a good boundary condition even over a range of 100 keV and that the distant level term cannot be ignored in the compound nucleus cross section. Such terms cannot be obtained by the average of a one level Breit-Wigner formula, as done in Ref. 3. A multilevel average, including the effect of distant levels, as per-

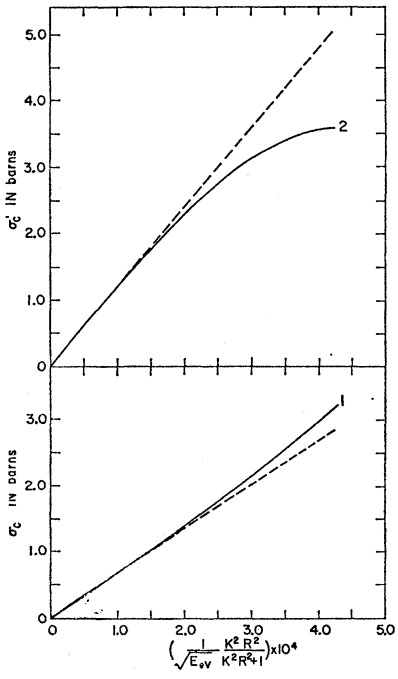

FIG. 6. The calculated P-wave compound nucleus cross section for the parameters of Fig. 5 in the range of 10 to 100 keV of neutron energy. Curves 1 and 2 are for the values of V_0 and W as (50.7, 2.38) MeV and (54.8, 2.10), respectively. The solid lines are the calculated curves and the dotted lines are the extrapolations from low energy, assuming a straight line behavior.

[19] E. J. Campbell, H. Feshbach, C. E. Porter, and V. F. Weisskopf, Tech. Rep. 73, 1960 (unpublished).

formed by Lane and Thomas in Ref. 1, can give rise to such terms.

The p-wave strength functions (in units of 10^{-4}) corresponding to curves 1 and 2 of Fig. 6, calculated from the slope of the curves in the low-energy limit, are 5.41 and 9.76, respectively. This shows that with the proper inclusion of the higher order terms one gets two values of p-wave strength functions, differing by about 80% in this case.

Nb^{93} falls at about half-maximum on the p-wave giant resonance curve for the parameters chosen. The p-wave contribution dominates the total cross section at energies of interest. The p-wave giant resonance being symmetrical about the center, one can get two values of V_0 (other parameters being kept fixed) which will give the same total cross section at a fixed nuetron energy, if there is no contribution due to $R_\infty{}^l$. The p-wave strength function or the compound nucleus cross section for the two cases will be the same. On the other hand, because of the comparatively large contribution of $R_\infty{}^l$ near the half-maximum of the resonance curve, one gets two pairs of values of the p-wave strength function and $R_\infty{}^l$ giving almost the same total cross section.

There are two sets of parameters that fit the data if only V_0 and W are varied and other parameters are kept fixed. If other parameters like R and a are also varied, then one can get other fits as well, which correspond to different values of the strength function $R_\infty{}^l$. The effect, in general, depends on the atomic weight and increases rapidly with increasing energy.

V. CONCLUSION

It is shown that the simple linear relation between the compound nucleus cross section and the strength function is the low-energy approximation to the more general formulation of Wigner and Eisenbud, as presented by Lane and Thomas. Such an approximation may not be valid near the giant resonance maxima and at high energies (>30 keV). In the calculation of Refs. 3 and 5 the approximate result is thought to be due to inadequate treatment of the average of the distant and nearby levels.

The boundary condition parameter $B_l=-l$ is shown to give the most reasonable form of the strength functions. For a diffuse potential of the Eckart form, the Lane and Thomas formulation can still be carried out even at high energies provided the matching radius is taken as about $R+(\frac{4}{3})a$, where R is the half fall-off point of the potential and a the diffuseness.

Even up to 100 keV the contribution of the nonlinear terms (including the distant level effect) in the p-wave compound nucleus cross sections are as much as 40% in the case of Nb^{93}. Because of the nonlinearity it is difficult to find a unique solution for the p-wave strength functions. There are two and only two values for a given radius and diffuseness. These values differ by about 80% in the case of Nb^{93} and correspond to values of $R_\infty{}^l$ which differ in sign. The effect will, in general, depend on the atomic weight and is a rapidly increasing function of energy.

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